

# The measurement errors in the *Swift*-UVOT and *XMM-OM*

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## ABSTRACT

The probability of photon measurement in some photon-counting instrumentation, such as the Optical Monitor (OM) on the *XMM-Newton* satellite, and the Ultraviolet/Optical Telescope (UVOT) on the *Swift* satellite, does not follow a Poisson distribution due to the detector characteristics, but a binomial distribution. For a single-pixel approximation, an expression was derived for the incident count rate as a function of the measured count rate by Fordham, Moorhead & Galbraith. We show that the measured count rate error is binomial, and extend their formalism to derive the error in the incident count rate. The error on the incident count rate at large count rates is larger than the Poisson error of the incident count rate.

**Key words:** instrumentation: detectors – methods: data analysis – methods: statistical – techniques: photometric.

## 1 INTRODUCTION

In recent years photon-counting detectors have come into operation in for example the Ultraviolet/Optical Telescope (UVOT; Roming et al. 2005) on the *Swift* gamma-ray bursts satellite, and the *XMM* Optical Monitor (OM; Mason et al. 2001). The microchannel plate intensified CCD (MIC) detectors used in these instruments have been discussed by Fordham, Moorhead & Galbraith (2000). These photon-counting detectors operate as follows: incoming photons exit electrons on a photocathode. The electrons are amplified by a stack of microchannel plates and then the amplified electron signal is converted back to a light pulse using a phosphor screen. Below this, a fibre bundle directs the light to a fast-scanning, frame-transfer CCD. After each frame is readout, the resulting charge events in the CCD are centroided by the on-board electronics.

At high incident fluxes, a photon-counting detector is limited due to coincident photon arrivals in a single readout of the detector. This represents a clear difference between the photon-counting technique and measurements made by direct illumination of a CCD, which can handle large fluxes, but has a higher background.

Normally, when measuring the number of counts arriving in a certain time interval, little further thought is given to the statistics of such a measurement, which were worked out long ago by Poisson (1838). Indeed, photon-counting instrumentation, like photomultiplier tubes, are usually seen as an exemplary case of Poisson statistics. However, due to the instrumental limitations imposed by centroiding and event detection of the MIC detectors, no more than a single event recording per pixel is possible in the smallest time-slice of measurement. This handicap prevents the full distribution of photon arrivals being sampled and thus the measurements are not

Poissonian, though the incoming photons follow a Poissonian distribution. As a result the errors on the photometry from the UVOT and OM do not follow Poisson statistics.

For each observation, however, one can derive the measurement statistics, which we show in Section 2 to follow a binomial distribution, and relate them to the Poisson distribution of the incident photons. Based on the measured distribution and the functional relation that it has to the incident Poisson distribution, we derive the errors in the measurement and in the inferred incident photon count rate in Section 2. This paper aims at providing the users of the UVOT, OM and similar instruments, a proper way to estimate the errors in their photometry.

## 2 THEORY

### 2.1 The mean number of incoming photons related to the measured count

For the detectors of interest, an exposure will be for a certain time period  $\Delta T$  and consist of  $N_f$  time-slices usually called ‘frames’. Exposing and reading out each frame takes a certain fixed time  $T_f = \Delta T/N_f$ , called the frame-time. Since during readout of the detector no incoming photons are detected, a fraction  $f_d$ , called the dead time, needs to be accounted for when determining the count rate.<sup>1</sup>

In the following, we will use variables for the total observation. For example, observed counts refer to all observed counts during the

<sup>1</sup> This is a simplification, since during the frame-transfer time, photons arrive, and charge is deposited in the CCD, they can be centroided into events when bright enough. The charge shunting process during frame transfer does lead to charge from a star (a fixed-position source) being ‘smeared’ out and this leads to readout streaks from bright stars.

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observation. This simplifies the treatment of the errors somewhat, and conversion to commonly used count rates and their errors is quite straightforward.

Now consider a single pixel. During an exposure  $N_f$  measurements are taken from that pixel, measuring either 0 or 1 count per frame, since coincident counts are recorded as a single event. It is here, where the difference with a Poissonian measurement comes in, since multiple detections in a single frame count only for one. We can use that fact to relate the probability of observing 1 or 0 photons to the fact that the incoming photons follow a Poisson distribution.

The Poisson probability that  $k$  incoming photons fall on one frame is a function of the mean incident counts per frame  $\mu$ :

$$P(k, \mu) = \frac{e^{-\mu} \mu^k}{k!}. \quad (1)$$

The first two moments of the Poisson distribution are  $\sum_{k=0}^{\infty} P(k, \mu) = 1$  and  $\sum_{k=0}^{\infty} k P(k, \mu) = \mu$ . The effective exposure time is less than the elapsed time due to the dead time. Therefore, the mean number of incoming photons  $C_i$  during the observation relates to the mean probability of measurement as  $\mu = C_i(1 - f_d)/N_f = \alpha C_i/N_f$ , where  $\alpha = (1 - f_d)$  has been introduced for notational convenience.

The measured number of photons in  $N_f$  frames, considering that for  $k > 1$  only one photon is counted, is

$$C_o = N_f[0 \cdot P(0, \mu) + 1 \cdot P(1, \mu) + 1 \cdot P(2, \mu) \dots]. \quad (2)$$

Using the equations above, this can be written as

$$C_o/N_f = 1 - e^{-\mu} = 1 - e^{-\alpha C_i/N_f}. \quad (3)$$

This functionally relates the incoming counts to the measured counts, and was originally derived by Fordham et al. (2000).

## 2.2 The error in the measured counts

We first show that the incident Poisson distribution leads to an observed binomial distribution due to the coincidence loss in the measurements, and then discuss the calculation of the measurement errors.

If we had an instrument that would be able to record the incoming photon distribution, the probability of recording  $m$  incident photons in  $N_f$  frames is given by the Poisson distribution. In actuality, not more than one photon can be measured per frame, so the distribution becomes modified in that term. Therefore, the probability of recording  $m$  incident photons in  $N_f$  frames is given by

$$\hat{P}(k; N_f, \mu) = \binom{N_f}{k} P(0, \mu)^{(N_f-k)} P(m > 1; \mu)^k, \quad (4)$$

where  $m$  reduces to  $k$  measured photons, since for each frame where  $m > 1$ , only one count is recorded.

Substituting  $k$  for  $m$ , using equation (3), and defining for convenience  $p = e^{-\mu}$  we can rewrite this as

$$\hat{P}(k; N_f, p) = \binom{N_f}{k} p^{(N_f-k)} (1-p)^k, \quad (5)$$

which is indeed a binomial distribution. That means that the observed counts are governed by a binomial distribution, and that errors need to be accounted for accordingly.

The observed error in the mean number of counts in the observation  $C_o$  for the binomial measured distribution will be determined by the binomial error

$$\sigma_o = \sqrt{C_o(N_f - C_o)/N_f}. \quad (6)$$

Using the observed error, the incident photon count rate error can be derived using the non-linear equation (3), because the relation has a 1–1 correspondence. Subtracting the mean count rate from the count rate with a  $1\sigma$  error added or subtracted, we obtain the following expression relating the upper and lower error  $\sigma_i$  in the incident counts to the error in the observed counts (see Fig. 1):

$$\sigma_i^+ = -\frac{N_f}{\alpha} \ln \left( 1 + \frac{\sigma_o}{N_f - C_o} \right), \quad (7)$$

$$\sigma_i^- = -\frac{N_f}{\alpha} \ln \left( 1 - \frac{\sigma_o}{N_f - C_o} \right). \quad (8)$$

For the highest incoming photon fluxes, the upper error becomes larger than the lower error, but for frame rates less than 0.9, the error is in a linear regime and they are nearly equal in absolute size.

## 3 DISCUSSION

### 3.1 Asymmetry

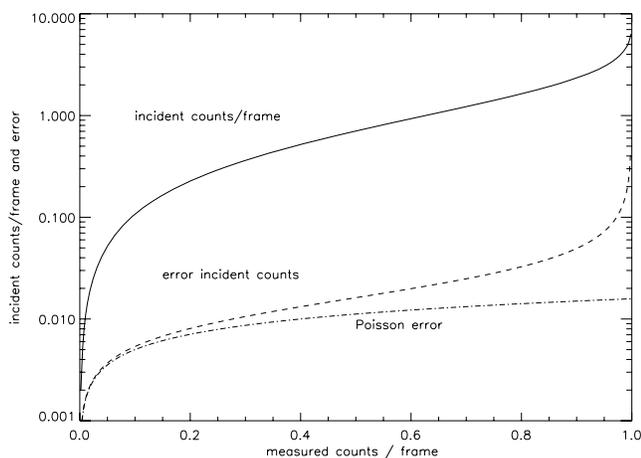
For a point source with a certain count rate, the incoming counts will fluctuate in a Poissonian sense around the mean. As discussed in Section 2, the measured counts are binomial. Because of this, the counts above the mean will be mapped into a smaller range of observed count rate than those below the mean, which is ultimately due to the coincidence loss. In this sense, the width of the distribution, as defined by  $\sigma_o$  is not an equal measure for the area under the distribution above and below the mean. We therefore need to be careful when interpreting the standard deviation derived here, especially for high-observed counts per frame values.

### 3.2 Mapping of the uncertainty range

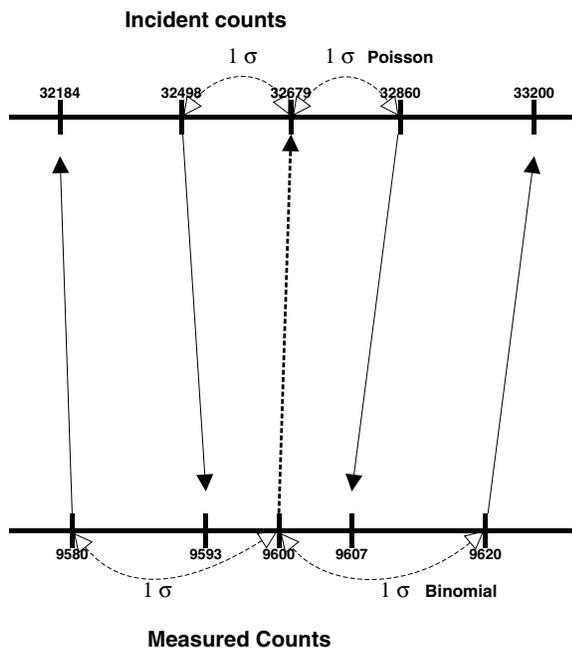
There is a certain inherent width in the distribution of incoming counts which results in Poissonian variation around the mean, usually expressed as the Poisson error. The question is how that error relates to the final error in the measurement.

In the limit of a small number of counts per frame, they become equal. For larger numbers of counts per frame they diverge, and the measurement error, after being mapped back to the uncertainty range in the incoming count rate, becomes dominant. Since the magnitude of this effect is not very apparent from the theory above, an example has been prepared in Fig. 2.

For simplicity, the number of observed counts has been set at  $C_o = 9600$  for  $N_f = 10000$  frames (which would be about a 110-s exposure in the default UVOT observing mode). The dead time is assumed to give  $\alpha = 0.985$ . Using the equations above, the incident number of counts is then  $C_i = 32679$ , with an associated Poisson error of 181 counts. In the figure we place the incident counts and its error on the top horizontal line. If we map the incident counts at  $\pm 1\sigma$  to the measured values they come out to be seven counts above and below the mean observed counts. The  $1\sigma$  binomial error on the observed counts, however, is 20, much larger than what would be expected from the mapped-back incident distribution. Mapping the measured counts at  $\pm 1\sigma$  from the measured counts back to the incoming counts, it is readily seen that these have a much larger spread than the incoming distribution. This effect becomes smaller for lower ratios of  $C_o/N_f$ . Please note that the values we chose for our example have a high coincidence loss which makes these effects more discernible.



**Figure 1.** The ratio of the incident count rate normalized to the frame time is shown as a function of counts per frame (=count rate/frame rate) along with its error (dashed), see equation (7). For comparison, the error in the Poisson limit has been plotted also. The assumed number of frames for error computation was 4000.



**Figure 2.** Schematic illustration of the effect of the coincidence loss on the errors. The top line represents the incident counts, the bottom line the measured counts. The error in the incident counts due to Poisson noise is indicated and how it projects to the measured counts. The binomial error on the measured counts has been indicated and how it maps to the incident counts.

### 3.3 Confidence levels

Confidence levels measure what percentage of the distribution of the measured quantity fall within certain limits. In a way they are more useful than the standard deviation in the presence of asymmetries, because they provide information on the reliability of the measurement. It is well known how to determine confidence levels for the measured count rate, because it follows the well-known binomial distribution (Gehrels 1986). However, the values reported are the incident count rates which bear a non-linear relation to the measured

ones. Likewise, a certain confidence level in the measured count rate will not imply the same level in the incident count rate, precisely because of the asymmetry mentioned above. The effect is largest at the highest count rates, where we showed by example above, that the measured distribution is much broader than the incident (Poisson) distribution. As a result, at high count per frame rates, the uncertainties in the measured count rate dominate those in the derived incident count rate. Also, in the limit of a low number of counts per frame the binomial confidence levels on the measured counts will approach the confidence levels of the Poisson-distributed incident counts because the distributions are identical in the low limit. The coincidence-loss correction at the limit of low counts is also negligible. This suggests that using the confidence limits for the measured binomial counts will be a good approximation for the confidence limits on the derived incident count rate.

### 3.4 Background

In general, for low count rates the effects from coincidence loss are negligible. This is especially true for the background. However, it was found that in some UVOT observations a correction for coincidence loss to the background was necessary and had an impact on the net source rates derived. Since the background is diffuse in nature, the arguments brought forward for considering the coincidence loss in diffuse situations by Fordham et al. (2000) need to be taken into account. They discussed this case in terms of the coincidence-loss area over which coincidence loss acts and the exposure area. Their equation reverts to the single pixel case for the background.

It is therefore important to realize that the expressions above, which were derived in the single-pixel approximation, need to be applied with caution to the background. If the measurement background area covers more than one CCD pixel, a normalization to the coincidence area, which is presumably one CCD pixel, needs to be made to apply the formulas above. For example, if a physical pixel has  $8 \times 8$  subpixels, the normalization is as follows. If  $C_B$  background counts were measured from a region of  $X$  subpixels,  $X$  larger than 64 subpixels, then the coincidence-loss correction for the background should be based on  $64C_B/X$  counts. In practice, the correction is not as firmly known as that because the centroiding may make the coincidence area larger or smaller. The UVOT FTOOLS software uses 78 subpixels which was chosen because that is close to the theoretical value and also the pixel area that was used to derive the empirical coincidence-loss correction (see Section 3.5).

### 3.5 The single-pixel approximation

The coincidence-loss formula under the single-pixel approximation has been very successful in predicting the correct rates in the UVOT (Poole et al. 2007). Other support for the use of the single-pixel approximation to calculate the coincidence-loss effect on the observed count rate comes from studies during the construction of the detectors (Fordham et al. 2000) and the implementation of the centroiding (Michel, Fordham & Kawakami 1997). The measurement algorithm locates the centre of the photon splash, which generally falls across two–three CCD pixels, and has an accuracy of a small fraction of a CCD pixel (allowing recording of UVOT and OM data with an accuracy of 1/8 of the physical CCD pixel size). Anomalies are rejected using four out of nine CCD pixels. As a result, the action of coincident photons is distributed over several pixels on the detector and are also folded through a screening algorithm. The net effect turns out to be a strengthening of the single pixel approximation, although the exact size of the coincidence-loss region, and its

relation to the physical CCD pixels, is still under study. Were the detections really independent single-pixel measurements, then it is easy to show that photon splashes which would fall in different ways over pixel boundaries would reduce the effects of coincidence loss by 10–20 per cent at high count rates. In reality, a small upwards empirical correction of the order of 6 per cent is found to be needed to the theoretical single-pixel rate in the UVOT (Poole et al. 2007) and OM, which is perhaps due to loss of some measurements of truly coincident, but slightly displaced, photons. Those could distort the symmetry of the electron splash on the detector sufficiently to be screened out as bad data.

### 3.6 Dead-time accounting

In the original formulation of the coincidence-loss correction (Fordham et al. 2000) the effects of the detector dead time in each frame were discussed but were not explicitly included in the coincidence-loss correction equation. As a result, early corrections for the coincidence loss did not include this term. Since the current formulation includes this term, no further correction for dead time is needed after application of equation (3).

### 3.7 Photometric packages

Currently most astronomical photometry software, like IRAF and DAOPHOT, may incorrectly report the error for measurements like these, because generally the assumption is made that the photometric measurements are dominated by Poisson noise. That is considered a good assumption for photomultiplier and normal CCD measurements. As we show in Fig. 2, the Poisson measurement error underestimates the error in these photon-counting instruments affected by coincidence loss.

## 4 CONCLUSIONS

We have shown in this paper how to derive the error in measurements made with photon-counting detectors of the type used in the *Swift* UVOT and *XMM* OM instruments. By comparing to the Poisson error usually used in photometry we make clear how significant this effect can be, and consider that users of these instrument must use our formalism to derive the errors in their measurements.

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